An extended abstract of PhD dissertation

"Valuation of market risk by Value at Risk method using copulas"

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1. Introduction

With the progress of globalization not only theoreticians and practitioners of economy but also other representatives of social sciences are dealing in economical practice with the term *risk* and *risk management*. Many contemporary scientists are perceiving risk as a threat for further worldwide development.

"… the world is no longer able to control risk created by modernity, and precisely: the faith is extinguishing present society will be able to control the threats caused by them – not because of neglects and failures of modernity but rather because of their victory (…). It becomes obvious that responses of individual countries to that global challenge are like reaction of Stone Age to the problems of the Industrial Age." These words come from Urlich Beck book "World Risk Society". The author draws attention to the fact that with increasing globalization in many areas of human activity, individuals have less and less impact to limit results of risk.

This observation is also valid for processes taking place in the economy. In the last few decades we witnessed the economic crisis which rose as a result of lack of control and limitation of the investment risk (e.g. bankruptcy of Barings Bank, the crisis on the mortgage credit market in the USA in 2007). The global character of financial market transactions is causing the effect that individual actions with too high risk level result in the higher risk on far away markets.

The history of using quantitative sciences for limiting risk of economic decisions has the beginning in the first part of 18th century. At that time the only mechanism of limiting risk were insurance contracts. Two hundred years later when derivatives started their career on the market the need for better construction of risk management methods has arisen. Nowadays many countries are taking steps for obliging their financial institutions to construct the proper procedures for protecting against too high risk exposure.

In the EU countries since few years two big projects are introduced. These are Basel II and Solvency II. The first one concerns the control and mitigation of risk in the banking system, the second one – in the insurance companies. Both projects are based on the assumption that undertakings cannot be forced to using one universal risk management system. The model should be tailored to the profile and structure of the undertaking's business. In the second pillar of Basel II the companies are even encouraged to develop their own models of market risk valuation using the method of Value at Risk.

The idea of risk measuring using the Value at Risk concept (*VaR*) is devoted to finding the amount of capital that is necessary to hedge the company against the high losses from the undertaken business in the assumed investment horizon. To set the value of this capital one needs arbitrarily set the investment horizon and the level of probability of the highest losses that can be accepted by the investor. Generally the legislation system sets the investment horizon to one workday and probability level not exceeding 1%.

Market risk of the investment covers the risk connected with the exchange rate changes, interest rate changes, equity price changes and commodities prices. Many factors influence the final level of this risk. One cannot assume that the above factors are independent so in the risk valuation method one should consider also the dependence structure between them.

In the last years the copulas are used to the description of the comprehensive dependence structure specification. They have an advantage over the typical measures of the degree of dependence (linear correlation coefficient, Kendall's τ, Spearman's ρ) because the copulas are not the single number but the function set on the space $\mathbb{R}^n$.

n-dimensional copula is n-variate distribution function on $\langle 0,1\rangle^n$ whose univariate marginals are uniformly distributed on $\langle 0,1\rangle$.

The usage of the copulas in practice needs to develop the method which on the basis of the empirical data would allow for the estimation of the copula's parameter or parameters. The common procedures of the estimation of this parameter(s) are:

- maximum likelihood method (MLE),
- canonical method of maximum likelihood ($CMLE$),
- inference function for margins,
- calibration method.

In the papers of (Tsukuhara, 2005), (Foscolo, 2008) and (Weiss, 2010) the minimum distance method of the copula parameter estimation was used. The results of the analysis lead to the conclusion that the minimum distance method can serve as the good way of copula parameter estimation. The development of the achieved results in this area seems to be advantageous. And additionally the measuring the effectiveness of the minimum distance method in the market risk management on the basis of Value at Risk seems to be very interesting.

2. THE AIMS AND THE THESIS OF THE DISSERTATION

The primal methodologies of the *VaR* estimation are based on the historical value of the undertaken investment analysis (historical method) or on the assumption that joined distribution of the variable combined from the risk factors is the normal distribution (covariance method and Monte Carlo (MC) simulation method). Above mentioned legal recommendations of own internal models of the *VaR* calculation give the opportunity to conduct the research in the area of Value at Risk calculation.

The main goal of the presented thesis is the construction of the method of Value at Risk estimation in such a way that the dependence among the risk factors are modeled using the copula function and the obtain values of are acceptable. The operation and effectiveness of the proposed method has been compared to the standard methods i.e. covariance method, historical method and MC simulation method.

The realization of the research required the testing of the following propositions:

- Market risk is the phenomenon that can be measured by the quantitative methods (measures of risk),
- Value at Risk is the risk measure,
- Value at Risk as the commonly used risk measure has drawback that it ignores the theoretical assumptions of itself,
- Legal recommendations encourage the financial institutions to using the value of risk concept despite its deficiency,

- There exist some measures which allow for comparison of the efficiency of different methods of *VaR* estimation,
- The copulas are the tool that can be effectively used to describe the dependency of two or many risk factors,
- The estimation of the copula parameter on the basis of the empirical data series and the selection of the optimal family of the copulas for gathered data may be done with the minimum distance method with the distance measure produced by the norm $\|x\|_p = \left(\sum_{i=1}^{k}|x_i|^p\right)^{\frac{1}{p}}$ from the space l^p where $p \geq 1$.

3. The research methods

Realization of the assumed aim of PhD dissertation and formulated above prepositions required to use several research methods.

The first part of the dissertation is theoretical and includes the overview of the literature connected with the main terms from the dissertation. These are:

- Definition of risk as a measurable category.
- Definition of risk measure and the analysis of the required properties of the risk measure.
- Definition, properties and methods of calculating Value at Risk.
- The analysis of the reasons why Value at Risk is used in economy practice.
- Showing the definitions and properties of copulas.
- The analysis of the copula parameter estimation methods and method of the optimal copula choice for the assembled data.

In the thesis the most important achievements in the area of copula parameter estimation methods, Value at Risk computing methods and applying copulas for estimating VaR are documented. In this part of the dissertation the research method are compared. The concepts of risk, risk measure and Value at Risk are not equally formulated. Therefore it is necessary to concern advantages and disadvantages of the most popular definitions of terms in the literature in order to define risk and Value at Risk to be in operation in the next part chapters of the thesis.

The second part of the dissertation is empirical and is devoted to make an assessment of effectiveness of copula parameter estimation methods with the usage of the minimum distance method. There is also empirical verification of the quality of the new proposed procedure for VaR computation method. In this part of the dissertation two research methods were used: experiment method (computer simulations) and statistical method.

The computer simulations are run with self-written procedures in the environments: R.3.1.2 and Mathematica 9. The large multi-dimensional samples from the distribution determined by the known family of copula and their parameter are simulated. In order to statistically analyze the data and make a statistical description and verification of simulated samples many statistical indexes and tests are used, e.g.:

- bias and mean square errors of estimators,
- Shapiro-Wilk test, Kupiec test and Christoffersen test,
- loss function[1] for the Value at Risk.

4. Structure and description of the study

The proposed in the previously described points research thesis and methods are viewed in the dissertation structure. It is composed of five chapters among which the first four ones are theoretical and the last one is focused on empirical analysis. The dissertation contains also the introduction, the summary, the bibliography and the appendix. The appendix presents self-made tables and figures which are illustrations of the indirect research results but not so important to put them into particular chapters.

The first chapter of the PhD thesis introduces the reader in the nuances of the term risk. Besides the historical outline of that term different types of risk (especially market risk which is the subject of dissertation) are described. Also axiomatic definitions of risk measure are here discussed.

[1] Two loss functions in the research were used: $L(VaR_\alpha^t(X), x_t) = \begin{cases} (x_t + VaR_\alpha^t(X))^2, & \text{for } x_t \leq -VaR^t(X) \\ 0, & \text{for } x_t > -VaR_\alpha^t(X) \end{cases}$ and

$$L(VaR_\alpha^t(X), x_t) = \begin{cases} \frac{(|x_t| - |VaR_\alpha^t(X)|)^2}{|VaR_\alpha^t(X)|}, & \text{for } x_t \leq -VaR^t(X) \\ 0, & \text{for } x_t > -VaR_\alpha^t(X) \end{cases}.$$

The second chapter concerns the review of risk measures. They are divided onto three groups: the neutral meaning of risk, the negative meaning of risk and sensitive measures. For each of these groups, if it is possible, the axiomatic definition from the previous chapter is assigned. The aim of these considerations is to legitimate why some risk valuation methods are used in economical practice whereas they do not have correct theoretical properties. Thus the term Value at Risk is ambiguous.

The third chapter includes the review of Value at Risk estimation methods. Some of them are strongly connected to the main interests of dissertation and they are described more precisely whereas the others are only signalized. In this part of PhD thesis different techniques of verification of VaR estimation methods are described. Additionally, the study of legal regulations which determine the usage of VaR for market risk valuation by financial institutions in Poland. On the basis of literature and earlier self-made researches one can distinguish the cases when the VaR measure does not fulfill the diversification property for portfolio.

The fourth chapter of the thesis is an introduction to the subject of copulas. After the copula definition has been chosen this chapter includes broad review of copula examples. In this part the ways of copula applications for computing Value at Risk are shown. There are also different methods of copula parameter estimation described.

The last chapter contains the empirical analysis. Firstly, there are results of two simulation researches with the usage of several copula families and main copula parameter estimation methods. These researches show that the minimum distance method can effectively estimate copula parameters. Although it is a time-consuming method it does not have disadvantages which are specific to other methods: complicated form of log-likelihood functions for some copulas and lack of opportunities to write the coefficients required in analytical formula (e.g. Kendall's τ for Gumbel-Barbett copula). Estimation with minimum distance method needs only the assumption that the formula of copula and the range of their parameter are known.

In the first part of the minimum distance method effectiveness test seven kinds of copula families with the dimensions $n = 2$ and $n = 3$ are concerned. For each of them five values of their parameters were randomly chosen. The values of bias and mean squared error were calculated on the basis of the results for samples counting $T = 50$ and $T = 100$ observations. In the second part of the test three families of the two-dimensional copulas were analyzed. The selection of these copulas results from including additionally estimation by calibration method and interval estimation method. The bias and mean squared error were calculated for the samples with $T = 250$ and $T = 500$ observations for forty different parameters of Clayton copula, Gumbel copula and Farlie-Gumbel-Morgenstern (FGM) copula.

In the second part of the fifth chapter one can find the description and results of the empirical analysis concerning the portfolio exposed to three risk factors in the period 05.07.2007-27.02.2014. Empirical data is here understood as the daily logarithmic returns calculated for exchange rate EUR/PLN, price of gold and WIG20 index from Warsaw Stock Exchange. In the research the Value at Risk in the one-day horizon was estimated on the basis of 250 observations from the preceding days. To describe it precisely let us denote one day of calculating VaR as t_0 so that the Value at Risk was estimated on the basis of data from periods $t = t_0 - 250, t_0 - 249, \dots, t_0 - 1$. In the presented research the estimators of VaR were calculated for 1432 days assuming the level of tolerance $\alpha = 1\%$ or $\alpha = 5\%$ and using:

- covariance method,
- MC simulations method,
- historical method,
- simulation method with the Clayton copula, Gumbel copula, Gumbel-Barbett copula, Frank copula, Farlie-Gumbel-Morgenstern copula, Ali-Mikhail-Haq copula and generalized Clayton copula.

Every situation when in the real daily data the loss of the portfolio exceeded the calculated VaR is called the exception. The number of exceptions was summed up for every method of VaR estimation in the test period (1432 days).

Described in this part of the PhD thesis the scheme of calculation of VaR estimator with the usage of simulation method with the above mentioned copulas is the realization of the main research goal of the thesis.

It should be stressed that the value added by the thesis is the usage of minimum distance method with the distance measure produced by the norm from the l^p space in all the tests described in the last chapter of the thesis for the extended scope i.e. for $p \in \{1; 1{,}5; 2; 4; 8\}$ and $p \to +\infty$, contrary to the previous research led only for $p = 1$, $p = 2$ and $p \to \infty$.

5. THE RESULTS

The proposed researches concerning the comparison of different copula estimation parameter methods gave the following results.

- According to the bias criterion and together with the increase of the length of the sample T the minimum distance method is more effective than calibration method and CMLE.
- CMLE should not be used for Gumbel copula parameter estimation because of crucially higher level of bias and MLE than other methods.
- Interval estimation is not an effective method of copula parameter estimation.
- The best results for minimum distance method are obtained when $p \approx 1{,}5$.

In the empirical analysis concerning computing VaR using copulas the following results are obtained.

1) The covariance method and MC simulation method have too high fraction of exceptions in the test period when tolerance level is $\alpha = 1\%$.
2) For tolerance level $\alpha = 1\%$ almost all methods of VaR valuation are acceptable because of Kupiec and Christoffersen statistics.
3) The historical method for both tolerance levels α has the fraction of exceptions in the test period the closest to α.
4) The hypothesis about the correctness of VaR estimation methods for $\alpha = 5\%$ should be rejected in case of CMLE and minimum distance method for Clayton, Gumbel, Frank and generalized Clayton copulas.
5) For the data used in the empirical research for VaR valuation the best result is obtained for Clayton copula with $p \to +\infty$ and $\alpha = 1\%$ and for FGM copula for $p = 8$ and tolerance level $\alpha = 5\%$. This choice is justified by using all the following criterions:

- fraction of exceptions in the test period is less than assumed tolerance level α,
- values of Kupiec test and Christoffersen test statistics are less than the proper critical values,
- the lowest value of the loss function in the test period for all concerned methods of VaR computation.

6) The analysis of distance function values for $p \in \{1; 1,5; 2; 4; 8\}$ i $p \to +\infty$ in the minimum distance method showed that the optimal family of copulas for the data is FGM copula.

6. Final results

The methods of VaR estimation with simulations using copulas are effective and often better than standard methods of Value at Risk valuation. The minimum distance method is a good way of copula parameter estimation. The empirical analysis does not confirm the hypothesis that value of parameter p should be close to 1.5 when MDE is used.

The presented paper is showing several possibilities of continuation further analyses connected to the problem of VaR estimation by copulas. The empirical research can be extended in the direction of usage other data series e.g. taking more factors of risk, introducing derivatives to the portfolio and also taking data from other markets than the Polish market.

BIBLIOGRAPHY

Acerbi, C. i Tasche, D., 2001. *Expected Shortfall: a natural coherent alternative to Value at Risk,* [Online] Available from: http://www.bis.org/bcbs/ca/acertasc.pdf [Accessed: 12.10.2012].

Albrecht, P., 2003. Risk measures. *SonderForschungsBereich504,* No. 03-01, *Contribution prepered for: Encyclopedia of Actuarial Science John Wiley & Sons,* [Online] Available from: http://www.ime.usp.br/~rvicente/risco/albrechts.pdf [Accessed: October 10, 2012].

Artzner, P. i in., 1999. Coherent measures of risk. *Math. Fin 9 (3).*

Bałamut, T., 2002. *Metody estymacji VaR,* Warszawa: NBP.

Beck, U., 2012. *Społeczeństwo światowego ryzyka. W poszukiwaniu utraconego bezpieczeństwa.* Warszawa: Wydawnictwo Naukowe SCHOLAR.

Bernstein, P. L., 1997. *Przeciw bogom. Niezwykłe dzieje ryzyka.* Warszawa: WIG-Press.

Best, P., 2000. *Wartość narażona na ryzyko: obliczanie i wdrażanie modelu VaR.* Kraków: Oficyna Ekonomiczna: Dom wydaw. ABC.

Biecek, P., 2008. *Przewodnik po pakiecie R.* Wrocław: Oficyna Wydawnicza GiS.

Bouye, E. i in., 2000. *Copulas for Finance. A Reading Guide and Some Applications,* [Online] Available from: http://thierry-roncalli.com/download/copula-survey.pdf [Accessed: October 12, 2012].

Breymann, W., Dias, A. i Embrechts, P., 2003. Dependence Structures for Multivariate High–Frequency Data in Finance. *Qantitative Finance 3*, s. 1-14.

Cherubini, U. i. in., 2004. *Copula method in finance.* Chichester: John Wiley and Sons.

Doman, R., 2011. *Zastosowania kopuli w modelowaniu dynamiki zależności na rynkach finansowych.* Poznań: Wydawnictwo Uniwersystetu Ekonomicznego w Poznaniu.

Durrleman, V. i in., 2000. *Which copula is the right one?,* France: Groupe de Recherche Operationelle Credit Lyonnais, [Online] Available from: http://www.ressources-actuarielles.net/EXT/ISFA/1226.nsf/0/9ef115c2bc569a7dc1257788002ae848/$FILE/copula-choice.pdf [Accessed: October 12, 2012].

Dys, J., 2010. *Dominacja stochastyczna a użyteczność.* [Online] Available from: http://apps.eui.eu/Personal/Researchers/JoannaFranaszek/prace/dominacja_stochastyczna.pdf [Accessed: April 16, 2015].

Embrechts, P., Lindskog, F., McNail., A., 2001. *Modeling Dependence with Copulas and Applications to Risk Management.* Zurich: Department of Mathematics, [Online]. Available from: https://people.math.ethz.ch/~embrecht/ftp/copchapter.pdf [Accessed: October 12, 2012].

Embrechts, P., McNeil, A. i Sraumann, D., 2002. Correlation and Dependence in Risk Management: Properties and Pitfalls. W: M. Dempster, red. *Risk Management: Value at Risk and Beyond.* Cambridge: Cambridge University Press, s. 176-223.

Fischer, M., 2009. *Multivariate copula models at work,* [Online] Available from: https://www.rmetrics.org/files/Meielisalp2009/Presentations/Fischer.pdf [Accessed: October 12, 2012].

Foscolo, E. i in., 2008. *New Estimators for Copula-Base models,* [Online] Available from: https://ceumathematica.files.wordpress.com/2008/09/articulo.pdf [Accessed: October 12, 2012].

Franke, J., Hardle, W. i Hafner, C., 2004. Copulas and Value at Risk. W: J. Franke, W. Hardle i C. Hafner (red.), *Statistics of Financial Markets.* New York: Springer, s. 301-310.

Gątarek, D. i in., 2001. *Nowoczesne metody zarządzania ryzykiem.* Warszawa: WIG-Press.

Genest, C., Quessy, J.F. i Remillard, B., 2006. Goodness-of-fit Procedures for Copula Models Based on the Probability Integral Transformation. *Scandinavian Journal of Statistics*, s. 1-30.

Hałaj, G., 2008. Przegląd metod badania płynności banków. *Bank i Kredyt*, lipiec, s. 14-27.

Heilpern, S., 2007. *Funkcje łączące.* Wrocław: Wydawnictwo Akademii Ekonomicznej im. Oskara Langego.

Hull, J. C., 2011. *Zarządzanie ryzykiem instytucji finansowych.* Warszawa: Wydawnictwa Profesjonalne PWN.

Jajuga, K. i in., 2007. *Zarządzanie ryzykiem.* Warszawa: Wydawnictwo Naukowe PWN.

Jajuga, K. i Jajuga, T., 2005. *Inwestycje :instrumenty finansowe, ryzyko finansowe, inżynieria finansowa.* Warszawa: Wydawnictwo Naukowe PWN.

Jakubowski, J., 2004. *Modele rynków finansowych.* Warszawa: SCRIPT.

Janicki, A. i Izydorczyk, A., 2001. *Komputerowe metody z modelowaniu stochastycznym.* Warszawa: Wydawnictwa Naukowo-Techniczne.

Jaworski, P. i Micał, J., 2005. *Modele matematyczne w finansach i ubezpieczeniach.* Warszawa: poltext.

Joe, H., 1997. *Multivariate Models and Dependence Concepts.* New York: Chappman & Hall/CRC.

Joe, H. i Xu, J. J., 1996. *The Estimation Method of Inference Functions for Margins for Multivariate Models,* University of British Columbia: Department of Statistics.

Jorion, P., 2000. *Value at Risk. The new benchmark for managing financial risk.* New York: McGraw-Hill.

Kijima, M. i Ohnishi, M., 1993. Mean-Risk Analysis of Risk Aversion and Welath Effects on Optimal Portfolios with Multiple Investment Opportunities. *Annals of Operations Research*, Issue 45, s. 147-163.

Kpanzou, T., 2007. *Copulas in statistics,* Stellenbosch: University of Stellenbosch, [Online] Available from: http://users.aims.ac.za/~tchilabalo/Kpanzou_Copulas_2007.pdf [Accessed: October 12, 2012].

Kupiec, P., 1995. Techniques for Veryfying the Accuracy of Risk Management Models. *Journal of Derivatives*, s. 73-84.

Luenberger, D. G., 2003. *Teoria inwestycji finansowych.* Warszawa: Wydawnictwo Naukowe PWN.

Łochowski, R., 2010. *Zmienność i jej ilościowe ujęcie za pomocą miar wybranych instrumentów finansowych w okresie kryzysu.* Warszawa, Akademia Leona Koźmińskiego.

Markowitz, H., 1952. Portfolio selection. *Journal of Finance*, Tom (7) , s. 77-91.

Matkowski, P., 2006. *Zarządzanie ryzykiem operacyjnym.* Kraków: Oficyna Ekonomiczna: Wolter Kluwer.

Mendes, B., de Melo, E. i Nelsen, R., 2007. Robust fits for copula models. *Communications in Statistics - Simulation and Computation*, s. 997-1017.

Mentel, G., 2011. *Value at Risk w warunkach polskiego rynku kapitałowego.* Warszawa: CeDeWu.

Nelsen, R. B., 2006. *An introduction to copulas.* 2nd ed. red. New York: Springer.

Niewęgłowski, M., 2001. *Value-at-Risk z uwzględnieniem ryzyka kredytowego,* Warszawa: praca magisterska na wydzile MiNI PW.

Olson, D. L. i Wu, D. 2010. *Enterprise Risk Management Models.* Berlin-Heidelberg: Springer.

Papla, D. i Piontek. K., 2003. *Zastosowanie rozkładów alfa-stabilnych i funkcji powiązań (copula) w obliczaniu wartości zagrożonej (VaR),* Wrocław: Akademia Ekonomiczna.

Pedersen, C. S. i Satchell, S. E., 1998. An Extended Family of Financial-Risk Measures. *The Geneva Paper on Risk and Insurance Theory*, Issue 23, s. 89-117.

Piontek, K., 2007. W: P. Chrzan, red. *Matematyczne i ekonomteryczne metody oceny ryzyka finansowego.* Katowice: Wydawnictwo Akademii Ekonomicznej w Katowicach, s. 113-124.

Pollatsek, A., 1970. A Theory of Risk. *Journal of Mathematical Psychology*, 7, s. 540-553.

Rockafellar, R. T., Uryasev, S. i Zabarankin, M., 2006. Generalized deviations in risk analysis. *Finance and Stochastics*, 10, s. 51-74.

Rockafellar, R., Urysaev, S. i Zabarankin, M., 2001. Deviation measures in risk analysis and opitimization. *Research Report #2002-7*, Gainesville: Department of Industrial and Systems Engineering University of Florida.

Rouvinez, C., 1997. Going Greek with VaR. *Risk 10*, s. 57-65.

Stryjek, A., 2005. *Jak mierzyć ryzyko instrumentów finansowych? Podejście aksjomatyczne,* Warszawa: Szkoła Główna Handlowa.

Stryjek, A., 2007. *Value at Risk a dywersyfikacja ryzyka,* Warszawa: Szkoła Główna Handlowa.

Stryjek, A., 2008a. *Efekt dywersyfikacji a efektywność klasycznych metod wyceny wartości zagrożonej (VaR) dla portfeli akcji GPW,* Warszawa: Szkoła Główna Handlowa.

Stryjek, A., 2008b. *Zastosowanie kopuli do szacowania wartości zagrożonej,* Warszawa: Szkoła Główna Handlowa.

Stryjek, A., 2009. Zastosowanie miar zalezności zmiennych losowych oraz kopuli Claytona i Gumbel-Hougaarda do szacowania wartości zagrozonej. *Przegląd Statystyczny*, s. 67-80.

Stryjek, A., 2011. *Porównanie wybranych metod estymacji parametru rodziny kopuli,* Warszawa: Szkoła Główna Handlowa w Warszawie.

Tarczyński, W. i Mojsiewicz, M., 2001. *Zarządzanie ryzykiem.* Warszawa: PWE.

Tsukahara, H., 2005. Semiparametric estimation in copula models. *The Canadian Journal of Statistics*, s. 357-375.

Uniejewski, P., 2004. *Koherentne miary ryzyka,* Wrocław: Politechnika Wrocławska.

Von Neumann, J. i Morgenstern, O., 1944. *Theory of games and economic behaviour.* New York: Princeton University Press.

Weiss, G. N. F., 2010. Copula parameter estimation: numerical considerations and implikations for risk management. *The Journal of Risk*, s. 17-53.

Winkler-Drews, T., 2009. *Zarządzanie ryzykiem zmiany ceny.* Warszawa: PWE.

Wolfowitz, J., 1953. Estimation by the minimum distance method. *Annals of the Institute of Statistical Mathematics*, Tom 5, s. 9-23.

Zagst, R., 2000. *Interest-rate management.* Berlin: Springer.